A

Synopsis

On

"COSMOLOGICAL SOLUTIONS OF MODIFIED THEORIES OF GRAVITY"

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Title: COSMOLOGICAL SOLUTIONS OF MODIFIED THEORIES OF GRAVITY

1. Introduction:

Einstein theory of gravitation was very much successful for constructing a cosmological model and explaining the origin and evolution of the universe. But Einstein theory fails to explain late time acceleration of modern cosmology. So many attempts have been made to modify the gravity theory which is to gratify the present accelerated phase. A natural generalization is to choose a more general action in which the standard Einstein-Hilbert action is replaced by an arbitrary function of the Ricci scalar R (Nojiri and Odintsov 2003b) (i.e, *f (R))* and is the name applied to *f (R)*-gravity. This modified theory may point this late time cosmic acceleration (Carroll *et al.* 2004). These *f (R)* models can satisfy local tests and integrate inflation with dark energy. For a brief review of *f (R)*-gravity one may refer to Nojiri and Odintsov (2003b), Carroll *et al.*(2004) and Capozziello *et al.* (2003). This modified gravity has recently been verified to explain the late time accelerated expansion of the Universe. One of the interesting and prospective versions of modified gravity theory is the *f (R, T)* gravity proposed by Harko *et al.* (2011) where, the gravitational Lagrangian is given by n arbitrary function of the Ricci scalar R and the trace of the stress-energy tensor T. They have obtained the gravitational field equations in the metric formalism as well as the equation of motion for test particles, which follows the covariant divergence of the stress-energy tensor. Akta¸ s and Aygün (2017) have also discussed Magnetized strange quark matter solutions in *f (R, T)* gravity with a cosmological constant and it proves that the f(R, T) gravity models can explain the late time cosmic accelerated expansion of the Universe.

An alternative approach to accommodate the current accelerating expansion of the universe is to modify the GR on large scale, such as, the scalar-tensor theories, f(R) theory, f(T) theory and DGP braneword theory (Yang 2011a; Zheng and Huang 2011) etc. Among these theories, the generalized teleparallel theory of gravity has recently gained a lot of interest due to its possible explanation about DE. In this generalization, the torsion scalar, T, is replaced by its general function f(T) in the Lagrangian of teleprallel gravity (Sharif and Rani 2011a). The f(T) gravity models use the Weitzenb¨ock connection which inherits only torsion and responsible for the accelerating expansion of the universe (Myrzakulov 2011). This approach (theory) originally developed by Einstein in 1928 under the name ”Fern-Parallelismus” or ”distant Parallelism” or ”teleparallelism” (Linder 2010; Unzicker and Case 2005). The f(T) theory has significant advantage of its second order field equations as compared to f(R) theory with fourth order field equations (Sharif and Shamir 2009; Sharif and Kausar 2010, 2011a; Jamil et al. 2012a, 2012b).

Bengochea and Ferraro (2009) described the recently detected acceleration of the universe without DE and performed observational viability tests by using recent SNIa data for some f(T) models. Linder (2010) explored that the cosmological constant is not only the component for the acceleration of the universe. This was also observed through a generalization of GR to other gravity theories. Zhang et al. (2011) discussed the dynamical analysis for the logarithmic and power form of f(T) models. They investigated phase space analysis of models, their stability of the critical points and evolution of 2 EoS parameter. In a recent paper. They also derived EoS parameter for two modified teleparallel models. Yang (2011b) investigated three new f(T) models and described their physical implications and cosmological behaviour. With the help of available observational data, some physical characteristics will be studied through the cosmological parameters, which are in good agreement with the observationally obtained present facts of the universe.

2. Review of Literature and development in the subject:

One of the predictions from inflation is that the observed universe should nearly isotropic on large scales. Spatially homogenous and isotropic universe can be well described by FRW model which cannot be explained the early universe. The existence of anisotropy at early times is a natural phenomenon to investigate the local anisotropies that we observe today in galaxies and cluster. Recent measurements on the cosmic microwave background radiation (CMBR) through WMAP suggest that the anisotropic inflationary models should be considered (Vielva 2004; Costa 2004). In early epoch of the big bang, the universe does not maintain its isotropic behavior at very small scales (Kumar and Singh 2008). In order to gets a realistic model representing an expanding, homogenous and anisotropic universe, Bianchi type cosmological models are considered. The isotropic behaviour of today universe makes BI model a prime candidate for describing the possible anisotropic effects of the early universe on modern day data observations.

Many people worked on simplest anisotropic model called BI universe to discuss the effects of anisotropy in several contexts. Kumar and Singh (2007) developed mechanism for solution of the field equations by considering BI universe model. The same authors (Kumar and Singh 2008) investigated the exact BI solutions which give the constant value of the deceleration parameter in scalar-tensor theory. Recently, Sharif and Waheed (2012) studied exact solutions for anisotropic fluids by taking the locally rotationally symmetric Bianchi Type I (LRS BI) universe model which generalizes the flat FRW universe in the modified theory (BD). There are many papers available in literature where this model has been used widely (Bali and Kumawat 2008; Amirhashchi 2011; Yadav and Saha 2012).

Recently, Bamba et al. (2011) discussed different f(T) models to investigate the cosmological evolutions of EoS parameter for DE and their observational constraints. Wu and Yu (2011) proposed two new f(T) models and analyzed that the crossing of

the phantom divide line is consistent with recent cosmological observational data. Chen et al. (2011) investigated this problem by extending modified gravity at the background and perturbed level. They also explored this theory for quintessence scenario.

Recently, Samanta and Myrzakulov (2017) have studied imperfect fluid cosmological model in f (R, T) gravity, Samanta et al. (2017) have studied KaluzaKlein bulk viscous fluid cosmological models and the validity of the second law of thermodynamics in f (R, T) gravity. Also Keskin (2018), Sharif (2018), Chaubey and Shukla (2013), Samanta (2013a, b), and Reddy et al. (2012, 2013) have studied cosmological models in f (R, T) gravity in different Bianchi type space-times. Presently, Bianchi universes are playing important role in observational cosmology, since the WMAP data (Nojiri et al. 2008; Bamba et al. 2012) seem to require an addition to the standard cosmological model with positive cosmological constant that bears a likeness to the Bianchi morphology (Jamil et al. 2012a, b; Steigman & Turner 1983; Hinshaw et al. 2009). Katore et al. (2015) have discussed domain walls in f (R, T) theory. Agrawal and Pawar (2017a, b) have studied Bianchi type-V universe model with magnetized domain walls in f (R, T) theory. Also, Agrawal and Pawar (2017a, b) obtained plane symmetric cosmological model with quark and strange quark matter in f(R, T) theory of gravity. Mostly the expansion of the universe is described within the framework of the homogeneous and isotropic Friedman–Robertson–Walker (FRW) cosmology. The reasons for this are purely technical. The simplicity of the field equations and the existence of analytical solutions in most of the cases have justified this over simplification for the geometry of space-time. However, there are no compelling physical reasons to assume the former before the inflationary period. To drop the assumption of homogeneity would make the problem intractable, while the isotropy of the space is something that can be relaxed and leads to anisotropy. Several authors (Jaffe et al. 2005, 2006; Campanelli et al. 2006, 2007) have studied particular cases of anisotropic models and found that the scenario predicted by the FRW model stand essentially unchanged even when large anisotropies were present before the inflationary period. Thus, the anisotropic Bianchi models become interesting in the academics. On this basis of this data, in the present study, Bianchi type-V space-time is taken into consideration. f(R, T) theory has attracted a lot of attention for the astrophysicist in recent times because of its ability to explain a lot of issues in cosmology and astrophysics (Barrow & Turner 1981; Sahoo et al. 2016a; Gron 1983a, b; Yousaf et al. 2017). Agrawal and Pawar (2017a, b) studied the f(R, T) theory in Bianchi Type V model. Pawar et al. 2016 also have analyzed two fluid axially symmetric cosmological models in f(R, T) theory of Gravitation.

3. Aim and Objectives of Research:

In recent years general relativity and its application to cosmology and cosmological models have been an interesting area of investigation. Cosmology deals with the study of large-scale structure of the Universe. The Universe consists of stars galaxies cosmic rays and background radiation. We live in this Universe and hence it is very important to know about the origin, the shape, the fate and the physical structure of the Universe.

We will promise framework for the understanding of these features and studying their physical behaviour. For that, we will construct mathematical models of the universe with the help of modified theories of gravity such as Scalar-tensor theory, Brans-Dicke theory, Gauss Bonnet gravity i.e. f(G) gravity, f(R), f(R,T), f(R,G), f(T), energy-momentum squared gravity theory i.e. $f(R,T_{\mu\nu}T^{\mu\nu})$ gravity, etc. We will derive cosmological solutions by considering all types of space-times including Bianchi types, Plane symmetric, Kantowski-Sachs, FRW space time and Kasner space-times. We will obtain the exact solutions of the field equations with perfect fluid, bulk viscous fluid, string, domain wall, Strange quark matter, etc. We will study some physical parameters of the models: geometrically as well as mathematically. Through our work we will try to focus light on the present and future aspects of the Universe. We will try to discover some unknown properties of the Universe which haven't known yet.

4. Methodology:

Knowledge of the concern topic will be obtained by the survey of research papers, review articles, various books and journals. Also we would like to visit different research institutes like IIT's, TIFR, IUCAA Pune, BHU Varanasi, IISC, SGBA Uni. Amravati, RTM Uni. Nagpur etc to interact with eminent Mathematicians and Physicist also would like to attend national, international conference related to our research work.

5. Importance of study:

Basic aim of our study is to determine the large-scale structure and evolution of the universe. This will be possible by constructing cosmological models in modified theories of gravitation. Universe, it is well known, is described by isotropic and homogeneous FRW models. However, experiments revelled that at the early stages of evolution of universe might not have been homogeneous and isotropic but anisotropies existed in the universe. Hence to study the behaviour and evolution of the early stages of the universe, i.e. immediately after the big bang, investigation of anisotropic models gained importance. Keeping in view of this fact, anisotropic axially symmetric and Bianchi type cosmological models with perfect fluid, domain wall, strange quark matter distribution in modified theories of gravitation will be done through our future work. Another basic role of this study is to understand the behaviour, shape and ultimate fate of the universe when sufficient observational data

and experimental results are available. Research in this direction is going on and we hoped that in near future definite conclusions will be arrived at.

6. Plan of Research Work:

Sr. No	Period	Plan of the work	Targets to be achieved
1	First year	Study of the background required for the research topic.	Good command on the subject and clear the course work.
2	Second year	Collecting the papers and analyze them.	Try to get some Construction techniques.
3	Third Year	Work on some problems to be included in the thesis.	Submit at least two papers.
4	Fourth Year	Publishing some papers and submitting the Thesis.	Some more papers and Thesis submission.

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